EXPOSITA NOTE

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Structure, clearinghouses and symmetry

Abstract We introduce and justify a taxonomy for the structure of markets and minimal institutions which appear in constructing minimally complex trading structures to perform the functions of price formation, settlement and payments. Each structure is presented as a playable strategic market game and is examined for its efficiency, the number of degrees of freedom and the symmetry properties of the structure.

1 An approach to money and credit

In this essay the stress is on the economics and the institutional aspects of trade. In a more technical companion essay (Smith and Shubik 2005) we relate these observations and the solutions to the strategic market games to phenomena encountered in physics. In particular we show that an important distinction in physics, between locally and globally defined symmetries, arises in the solution of market games in the same mathematical form. In all cases symmetries reflect the presence of apparent strategic degrees of freedom, which actually have no effect on allocations. When these are locally defined, it is possible for agents individually to factor them out of decision making, and arrive at the noncooperative equilibria of the game. When they are globally defined, they are under the control of noone, and the

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uncertainty they create for each agent can make it highly unlikely for the economy to arrive at any pure-strategy equilibrium.

1.1 Minimal mechanisms

We are concerned with the minimal level of complexity at which a phenomenon of interest appears. This relatively imprecise statement will be clarified in section 4 in examining the simplest mechanism for producing a market price.

2 The Jevons failure of the double coincidence of wants

Jevons (1875) provided a critical example illustrating the problems with barter. He considered three individuals trading in three commodities. He was able to select conditions to show that barter with utility improvement after each exchange was not sufficient to bring about optimal trade.

   In our analysis we contrast the generalized Jevons example with the situation in which all individuals own a general supply of all commodities. In our notation the starting allocation of commodity j to individual i will be a_i^j. We consider both the extended Jevons' example aI, which we denote by a vector a_{spec} and a more general endowment (a_i^j) denoted by a_{gen}.[1] We will generally consider m consumable goods to which m types of producers correspond in the specialist case. The total number of traders will be set to rm, with $r > 1$ traders of each type. An $(m + 1)$ good may be introduced when money is not one of the consumables.

3 Strategic market games and payment systems

In the past few years, one of us together with several colleagues (Shubik 1999) developed a set of games called strategic market games. They are fully formulated games which can be played, simulated and analyzed. They serve to contrast a game theory and gaming approach to the functioning of competitive markets for the exchange of individually owned economic goods and services with the general equilibrium analysis (see Debreu 1959). The stress in the formulation of strategic market games is upon the explicit formulation of the mechanisms for trade: in other words, upon the complete definition of the state space or the outcome set.

   A strategic market game for traders $i \in N$, with endowments $a_i \in \Omega^m$ and utility functions $u_i : \Omega^m \to R$, is defined by specifying the market structure M and the strategy sets S_i for each $i \in N_i$. Under fairly weak assumptions indicated below (Dubey, MasColell, and Shubik 1980), it can be shown that noncooperative equilibria exist for all of the class of games described above, where the participants in the markets use only simple messages. The conditions required are:

(A) Each u_i is concave and nondecreasing in each variable.
(B) $\sum_{i \in N} a_i > 0$.

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[1] If all $a_i^j = a$ and utilities are the same there will be no trade, but a shadow price can be established.

(C) For any $(j, k) \in M$, there exist at least two traders who have positive endowments of j and desire k, as well as two traders who have positive endowments of k and desire j. (A trader i is said to desire commodity j if u_i is a strictly increasing function of the jth variable).

The buy-sell game defined by (Dubey and Shubik 1978) is utilized here.

4 Degrees of freedom and symmetry in cash and credit markets

In our discussion here we limit our concern to simple one-period Cournot–Nash equilibria and observe how some of the set of the noncooperative equilibria may approach the competitive equilibria when the number of small agents becomes large.[2]

In the economics of money and financial institutions there have been several problems that have for the most part eluded careful modeling and have been treated primarily by verbal methods and historical commentary. They include, with some exceptions, items such as what difference does it make to the economy if different goods or instruments are selected as the numeraire; can a system operate with everyone being a banker and using their own IOU notes as a means of payment; are the bankruptcy and default laws institutional curiosities or do they represent logical necessities in a system designed to promote trade; how do the roles of a central bank differ or overlap with those of a money market; what is a merchant banker?

4.1 Functions and institutional forms

Mass economies with mass (more or less anonymous) markets require many functions to be performed in facilitating the completion of trade. Among the functions are:

1. Aggregation of bids and offers
2. Identification, verification and auditing
3. Credit evaluation
4. Record keeping
5. Insurance, storage and transportation
6. Credit granting
7. Clearance and payment
8. Final settlement and default resolution.

These functions are necessary parts of a financial system designed to facilitate trade in a dynamic economy involving both space and time.

In many economies these functions are supplied by a variety of institutions and individuals. For example there are individual traders, retailers, wholesalers,

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[2] Underlying all of the variations of the buy-sell market given in section 4 is the presence of an inactive equilibrium where no one trades. This is easily seen by observing that if all but one individual stay out of the market then the last individual has no motivation to enter. This is a highly implausible state, but formally fits the definition of a noncooperative equilibrium.

and non-financial firms. There are markets, credit evaluation agencies, banks, central banks and other government agencies, insurance companies, clearinghouses, accountants, lawyers, notaries and courts. At first glance the names on the list appear to be peculiarly institutional, yet the functions they perform are an integral part of defining the dynamics of trade.

In our analysis here we do not propose to dwell in any detail on all of these institutions, but we show that even with extremely stripped down models considered at a high level of abstraction several institutions are called for. In particular some form of credit evaluation agency, a clearinghouse, goods markets, a money market, the central bank and the courts must appear in relatively simple models.

4.2 Barter, commodity money, IOUs and clearing

In the shade of monetary theory there are a host of questions which from one point of view do not seem to be of central importance but are not quite covered in formal theory. By limiting ourselves to a single move game we are able to provide a formal structure to cover the many institutional differences in monetary systems. Yet, as is shown in Table 1 there are only a small number of structures that cover most of the essentials of monetary institutions found in the history of economics and finance.

In the Table 1 the salient features of eight market structures are illustrated. They are each considered below in some detail.

4.2.1 Barter

Barter involves bilateral trade between pairs of agents or coalitions. It describes a state before the existence of formal markets. As exchanges are postulated to be value given for value received there is no need for institutions. The emergence of markets and financial institutions from barter economies poses problems in economic anthropology, history and in the investigation of long term economic dynamics. These are all not dealt with here.

4.2.2 A commodity money: gold

In the buy-sell game of (Dubey and Shubik 1978), initial allocations to agent i are the vector

$$a_i = (a_i^1, \dots, a_i^m, a_i^{m+1}),$$

with Gold as the $(m+1)$th good. A strategy for a trader i is vector (q_i, b_i) of dimension $2m$ such that:

$$b_i^j \geq 0, \quad 0 \leq q_i^j \leq a_i^j \quad \text{for } j = 1, \dots, m \text{ and } \sum_{j=1}^m b_i^j \leq a_i^{m+1},$$

where b_i^j is the bid for and q_i^j is the offer of good j by i. A money is utilized to bid in all markets, while a nonmonetary good is offered for sale only in its own market. Thus, in general, a strategy for an individual i of type g is:

$$(b_i^1, q_i^1; \dots; b_i^m, q_i^m).$$

This has dimension of $2m$.

Table 1 The salient features of one-period exchange

	Barter	One commodity	Bimetallism	All goods as money (a)	All goods as money (b)	Money market and gold	(a) All IOUs with default	(b) All IOUs with clearing
Markets	Not defined	$m - 1$	$m - 2$	$m(m-1)/2$	$m(m-1)$	m	m	m
Numeraire	Free "ideal"	Gold	Gold linked to silver	Free "ideal"	Free "ideal"	Gold	Free "ideal"	Free "ideal"
Means of payment	All	Gold	Gold and silver	All	All	Gold	All IOUs	All IOUs
Store of value	All	Yes	Yes both	All	All	Yes	No	No
Agent types	1	2	3	1	1	2	1	1
Financial markets	No	No	No	No	No	Yes	No	No
Credit evaluation	No	No	No	No	No	No (variant yes)	No	Yes
Clearinghouse	No	No	No	No	No	Yes	Yes	Yes
Courts for default	No	No	No	No	No	Yes	Yes	No
Efficiency	Not always	Sometimes	Generically no	Generically no	Yes	Yes if enough money	Yes	Yes
DOF a_{spec}	Not defined	$2(m-1)$	$3(m-2)$	$m(m-1)$	$2m(m-1)$	m^2+m-1	m^2-1	m^2-1
DOF a_{gen}	Not defined	$2m(m-1)$	$2m(m-1)$	$m^2(m-1)$	$2m^2(m-1)$	$2m^2$	$(2m+1)(m-1)$	$(2m+1)(m-1)$

First line is the multiplicity of structures. The next four lines illustrate the four properties of money. The next four lines illustrate the emergence of financial institution function, and the last three lines cover efficiency and characteristics of exchange

In the special extended Jevons example a strategy is of the form

$$(b_i^1, 0; \ldots ; b_i^m, 0),$$

for the moneyed individual. This has dimension $m-1$, while the individuals without money each have a strategy of dimension of 1.

Even in this simple model a question concerning bidding and offering appears. It involves the possibility of wash sales. A wash sale occurs when an individual both buys and sells the same commodity thus creating the impression that the net market activity is greater than it is. Price formation for commodity j in general is given by:

$$p^j = \frac{\sum_{i=1}^m b_i^j}{\sum_{i=1}^m q_i^j}.$$

A minor item which requires comment is that we have not included the spot market of gold for gold. Such a market is not ruled out by logic, but will be inactive by virtually any type of optimizing solution. If individuals are assumed to act randomly or in error or under misperception it is possible to design games such as the "Dollar auction game" (Shubik 1971) which never should be played by an optimizing individual.

4.2.3 Bimetallism: an economy with both gold and silver as money

"The medium-of-exchange function can tolerate more than one money without too much trouble; the unit of account function cannot." (Kindleberger 1984, p. 55).

A well known old question in monetary theory has been the feasibility of using two or even three metals simultaneously as a means of payment. The usual physical argument is that individuals need to make three types of payment. Large payments to buy a house, for instance; middling payments to buy a bicycle, and small payments to buy a glass of beer. Gold is too valuable for the last two; silver fits best in the middle range and copper fits at the bottom.

Unfortunately if two or more commodities are used as a means of payment, and there is any change in the endowments of any commodity of value, as there almost certainly will be, the relative prices between the two moneys will change. If a country has open trade with others, silver or gold may flow in or out as a function of relative prices (see Kindleberger 1984, Ch. 4 for a nice summary). A central government will have to adjust the relative prices of silver and gold coinage if it wishes to keep both in circulation. In 1717 Sir Isaac Newton, Master of the Mint, observed that a lewidor (louis d'or) was worth 17s and 3f (f= farthing) in France, but 17s 6d in England, which brought a large inflow of gold to London (Kindleberger 1984, p. 54). Newton set the price of gold at $3 17s 10 1/2d in an attempt to adjust the ratio between gold and silver.

If we are concerned only with a one-period market with fixed endowments then an all-seeing government could do the appropriate calculations and announce a fixed mint price between gold and silver. Thus the number of free markets in an m commodity world would be reduced to $m-2$. At equilibrium, in this special case law, custom and free markets would coincide and there would be no net inflow or

outflow of gold or silver if there were no arbitrage opportunities. The static equilibrium theory shows that it is logically feasible for a country to impose an extra constraint by fixing the price between gold and silver. But the hope for running a dynamic economy without constant readjustment is negligible.

In terms of the buy-sell model the concept of a single money is well-defined strategically. If good m is the money it can be used to purchase all other goods directly. It enters the numerator of the price-formation mechanism for all the goods it purchases. Thus if an individual i of type m has as his initial endowment $(0, 0, \ldots, a)$ where the mth good is deemed to be the money, his strategy is of the form $(0, b_{im}^1, 0, \ldots, 0, b_{im}^{m-1})$ where

$$\sum_{j=1}^{m-1} b_{im}^j \leq a,$$

and $b_{im}^j \geq 0$. The bid has dimension $m - 1$.

Suppose however, that there are two monies, say gold and silver, the mth and the $m - 1$th commodities. For simplicity our remarks are confined to our closed one-period economy model. If a money is a means of payment but is not sold as a commodity then we can construct a well-defined playable one-period game for the extended Jevons example as follows.

An individual i of type m has as his initial endowment $(0, 0, \ldots, a)$ where the mth good (gold) is deemed to be the money, his strategy is of the form $(0, b_{im}^1, 0, \ldots, 0, b_{im}^{m-2})$ where

$$\sum_{j=1}^{m-2} b_{im}^j \leq a.$$

The bid has dimension $m - 2$. Similarly an individual i of type $m - 1$ (who owns silver, and has an endowment $(0, 0, \ldots, a, 0)$) has a bid of dimension $m - 2$.

A strategy of a trader of type g where $g = 1, 2, \ldots, m - 2$ is of the form $(0, 0; 0, 0; \ldots, q_{ig}^g, 0; \ldots; 0, 0)$ which is of dimension 1.

With the two monies how is price formed? We can specify generally for good j where $j = 1, 2, \ldots, m - 2$

$$p_j = \frac{f\left(\sum_{i=1}^k b_{i,m-1}^j, \sum_{i=1}^k b_{im}^j\right)}{\sum_{i=1}^k q_{ij}^j}.$$

If by government law a linear relationship in the valuation of gold versus silver has been set (for example around 1,700 the gold/silver ratio was in the range of 1–15 or 16) then the price formation is specified as:

$$p_j = \frac{\sum_{i=1}^k b_{i,m-1}^j + \alpha \sum_{i=1}^k b_{im}^j}{\sum_{i=1}^k q_{ij}^j}.$$

We now have a completely well-defined game whose solution will depend on the parameter α. It is here that law clashes with custom and free markets and context free mathematical economic models may mislead us away from institutional

understanding. If governments rule out by law market structures, black markets will spring up and ways to avoid the laws will be devised. The legal restrictions will become a cost of doing business and not a pure barrier.

In economic history as the relative prices of gold and silver moved, coins or ornaments were melted down and recast as ornaments or coin. This suggests that gold and silver could be considered as both monies and commodities for sale. If we were to consider them as both then a somewhat different model from the above can be considered. We may add in two extra markets. The market where the commodity, silver is sold for the money, gold and the market where the commodity, gold is sold for the money, silver. The strategy of an individual i of type m (those who own gold) now becomes of the form $(0, b_{im}^1, 0, \ldots, 0, b_{im}^{m-1}, q_{im}^m, 0)$ where

$$\sum_{j=1}^{m-1} b_{im}^j + q_{im}^m \leq a \quad \text{where } b_{im}^j \geq 0 \text{ and } q_{im}^m \geq 0.$$

The bid has dimension m. Similarly an individual i of type $m - 1$ (who owns silver) and has an endowment $(0, 0, \ldots, a, 0)$ has a bid of dimension m.

Can the government enforce the fixed rate between the two numeraires? If the government mint had a large supply of gold and silver and was willing to buy or sell in unlimited quantities at the rate it had set it could control the ratio. In these two models no quantitative intervention is modeled.

4.2.4 All goods a money? Case 1: money or goods?

In some writings (Clower 1967, for instance), it has been suggested that when all goods can be used as a means of payment then every good as a money is the equivalent of barter. This misses the important distinction concerning the existence or nonexistence of mass markets which produce a single price for many agents trading simultaneously through some form of aggregating/disaggregating mechanism (see Amir et al. 1990).

Constraining ourselves to the buy-sell game when all goods may be utilized as a means of payment we have complete markets. There is a market between every pair of goods. But generalizing from bimetallism, given the definition of a good and a money, in our extended Jevons example, if an individual has only one commodity, say oranges which may be used as a money to make purchases, can it also be sold as a good? If a good selected as a money can only be used for bidding then in the extended Jevons example nothing can be offered for sale as all individuals have only a money.

The game is completely symmetric, but requires one of two breaks with our treatment of all the other markets. If bids and offers retain a unique definition, so that the notation agrees with the other cases, then any individual i of type g has a strategy of the form: $(0, b_{ig}^1; \ldots; 0, b_{ig}^{g-1}; 0, 0; 0, b_{ig}^{g+1}; \ldots; 0, b_{ig}^m)$ This has a dimension of $m - 1$ and the only equilibrium is inactive. By definition only $m(m - 1)/2$ markets can be considered and none of them can go active because there are no goods for sale, there are only monies which can be bid. Alternatively, a new clearing rule with the same symmetry as the markets may be designed (Smith and Shubik 2005), which produces active and type-symmetric solutions, but without short sales these interior solutions can only be attained by two types.

4.2.5 All goods a money? Case 2: money and goods

If any money utilized for purchasing can also be used as a good for sale, then it is reasonable, in this instance to consider the existence of all $m(m-1)$ markets not $m(m-1)/2$. The owner of oranges buys apples in the oranges/apples market but can, if she wishes sell oranges in the apples/oranges market. There is no need to specify a numeraire. The game is completely symmetric and each individual has a strategy of $2(m-1)$ dimensions.

4.2.6 A commodity money and a money market

In an economy which employs a single commodity money such as gold without other money substitutes the equilibrium will be interior if there is enough money which is well distributed (the specific inequalities are given in Shubik 1999, Ch. 9). If the distribution of money is inappropriate, even if there is a sufficiency of money a boundary solution will exist. Efficiency can be improved by introducing a money market. A money market is a financial market. It requires a new instrument, the individual IOU note, and calls for both a collection agency and the courts to fully specify its functions under all contingencies. The price in the money market can be regarded as an endogenous rate of interest. It is formed by bidding IOU notes for a supply of money offered in the money market. In the extended Jevons example the rate of interest will be:

$$1 + \rho = \frac{\sum_{i=1}^{r} \sum_{g=1}^{m-1} z_{ig}^m}{\sum_{i=1}^{r} w_{im}^m},$$

where the z are the IOU notes for which the money is offered and w_{im}^m denotes the amount of commodity money an individual i offers for loan in the money market.

We must consider that the IOU notes are redeemed after trade. But it is possible that the economy could reach a state where an individual would not be in a position to redeem her IOUs. The default rules must be specified. The dimensions of the penalty will be utility/money.

The simplest game can still be defined with only one information set per individual. The extensive form has the individual borrow before bidding in the goods market. Realistically he would be informed before he bids, but this information can be finessed if we permit the individual to allocate percentages of his (unknown) buying power. In the extended Jevons example the strategy of a borrower is of the form: $(0, b_{ig}^1; 0, ...q_{ig}^g, b_{ig}^g; 0, b_{ig}^{m-1}; z_{ig}^m, 0)$ where $b_{ig}^j \geq 0$ and $z_{ig}^m \geq 0$. The strategy has dimension $m+1$. The strategy of a lender is of the form: $(0, b_{im}^1; 0, ...; 0, b_{im}^{m-1}; w_{im}^m, 0)$ which has dimension of m.

In this model as all transactions are for cash, all sellers can be paid directly from the market posts in cash, but then a collection agency must solicit each borrower for repayment and the courts must take care of any defaults. A variation of this model can have accurate credit evaluation which attaches a discount to each individual bid in a manner that avoids bankruptcy. We discuss this variation for the model in Section 4.2.8.

4.2.7 Everyone their own banker (a): all issue IOUs with no credit evaluation but default rules

Suppose that each individual is permitted to issue her own IOU notes as a means of payment. We may impose some arbitrary upper bound M on the amount of notes that any individual can bid. Furthermore we select an imaginary money "the ideal" as the numeriare, thus an individual IOU is a promise to redeem the paper in ideals. The utilitarian value of the ideal in default is established by the laws or rules of default.

We consider the extended Jevons example. A strategy by an individual i of type g, $g = 1, 2, \ldots, m$ is of the form $(0, b_{ig}^1; 0, \ldots, q_{ig}^g, b_{ig}^g; 0, b_{ig}^{m-1}; b_{ig}^m, 0)$. It is of dimension $m + 1$.

We develop two models. In the first model each market immediately ships all goods to all the bidders, but before it settles with the suppliers it sends all of the IOU notes to a clearinghouse together with its intended payments. The clearinghouse nets all IOUs along with the intended payments. If all net to zero, its work is done. The clearinghouse informs all individuals who are in default and all individuals who are creditors and it turns over these accounts to be settled by the courts.

The symmetry among all agents in this model is obtained by introducing the two societal agencies, the clearinghouse and the courts. In institutional fact the clearinghouse could be an agency of the government such as the Fedwire of the United States or it could be a privately owned institution such as clearing house interbank payment system (CHIPS). The courts should reflect their society, at any moment of time the laws are part of the rules of the game, but in a longer horizon they will be subject to change. The modeling and analysis of such a system is highly dependent on the time horizon considered. If only a few months or years are under consideration it is reasonable to accept the institutions as given. If several decades or centuries are being considered it makes sense to investigate the trade-off between law and custom.

4.2.8 Everyone their own banker (b): all issue IOUs with credit evaluation to balance the books

Suppose that, as in Case (a) above each individual is permitted to issue her own IOU notes as a means of payment. We may impose some arbitrary upper bound on bids as was done above. The strategy sets of the individuals are as before; but settlement rules are changed and we are able to introduce a credit evaluation agency to dispense with the courts and default rules by imposing a balancing of all accounts under all circumstances. This is done by utilizing the clearinghouse as both a clearing agency (see Sorin 1996) and a credit evaluator.

All markets receive rm different IOU notes in bids for the goods for sale. In order to make this game reasonably playable we envision that each market bundles all of the IOU notes and sends them to the central clearinghouse where they are all matched and netted. As they are all denominated in the Ideal which does not have a physical existence, the clearinghouse can impose a settlement rule by solving a simple linear system requiring the full balance of each individual by imposing a relative valuation of the notes. There are some technical problems with division by zero when individuals offer no goods whatsoever for sale, but nevertheless bid.

We may evaluate all bids accompanied with a zero offer as of zero value and 0/0 is interpreted as no trade.

4.3 A comment on the clearinghouse versus cash-in-advance

The explicit introduction of a clearinghouse is consistent with the no transactions costs aspects of general equilibrium theory. The difference being that we present a formal process oriented model where, in essence, the clearinghouse provides a zero interest loan in clearinghouse credit for the small period of time during which accounts are netted (see Summers 1994). The clearinghouse approach contrasts with the cash-in-advance models where a full time period lag is attributed to settlement.

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